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ENTROPY-TEMPERATURE AND TRANSMISSION DIAGRAMS FOR AIR

BY

C. R. RICHARDS

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 63 JANUARY, 1913

ENTROPY-TEMPERATURE AND TRANSMISSION DIAGRAMS FOR AIR

By C. R. Richards, Professor of Mechanical Engineering

CONTENTS

I. INTRODUCTION

1. Preliminary 3
2. Acknowledgment 3

II. CONSTRUCTION OF THE ENTROPY-TEMPERATURE DIAGRAMS

3. Values of dQ and $d\phi$ 3
4. Constant Volume Lines 4
5. Constant Pressure Lines 4
6. Isothermal and Adiabatic Lines 4
7. pv^n Lines 4
8. Intrinsic Energy 5
9. Variable Specific Heats 5

III. THE USE OF THE DIAGRAMS

10. Values of p, v and T for Isothermal Changes 6
11. Values of p, v and T for Adiabatic Changes 6
12. Values of p, v and T for pv^n Changes 6
13. Heat Changes 6
14. Graphical Representation of Heat Changes 7
15. Values for External Work 7
16. Expansion or Compression Along Irregular Paths 7
17. Isothermal Expansion and Compression 7
18. Adiabatic Expansion and Compression 9
19. The Work of the Complete Cycle 10
20. Expansion and Compression Along a pv^n Line 10
21. Heat Changes Along a pv^n Line 11
22. Flow of Air Through Nozzles 12

IV. USE OF THE CHARTS FOR GASES OTHER THAN AIR

V. THE AIR FLOW DIAGRAM

VI. MAXIMUM POWER TRANSMITTED THROUGH PIPES

ENTROPY-TEMPERATURE AND TRANSMISSION DIAGRAMS FOR AIR

I. INTRODUCTION

1. *Preliminary.*—In engineering calculations graphical methods not infrequently afford a satisfactory degree of accuracy, or when extreme precision is required, a check on analytical methods which prevents the possibility of arithmetical errors. Among the thermodynamical calculations which need to be frequently made, those dealing with the compression, expansion, and transmission of air and other gases are tedious and liable to error.

It is the purpose of this bulletin to present the theory and use of three graphical charts through whose aid all problems pertaining to compressed air may be quickly solved with a minimum of labor and with a degree of accuracy which is entirely satisfactory in engineering work. As with all graphical and mechanical aids, practice in the use of the charts is necessary to a full realization of their convenience and reliability.

2. *Acknowledgment.*—The original entropy-temperature diagram for air, Fig. 1, was designed by the writer several years ago. Recently, Mr. John A. Dent, Instructor in Mechanical Engineering, simplified the construction of this chart and improved it by the addition of a series of lines representing the curves of $pv^n = C$, as shown by Fig. 2. Fig. 5 is a diagram for the solution of problems on the flow of air in pipes, designed at the writer's suggestion by one of his former students, Mr. Walter J. Wohlenberg, and recomputed and redrawn for presentation here.

CONSTRUCTION OF THE ENTROPY-TEMPERATURE DIAGRAMS

3. *Values of dQ and dφ.*—As a result of the application of the fundamental principles of thermodynamics, there may be developed three general equations for the change of heat, dQ, for perfect gases. Of these, two are conveniently used in calculating the entropy change, thus

$$dQ = c_v dt + (c_p - c_v)\frac{T}{v}\,dv \quad \ldots\ldots (1)$$

$$dQ = c_p dt - (c_p - c_v)\frac{T}{p}\,dp \quad \ldots\ldots (2)$$

Since the change of entropy ($d\phi$) is represented by $\frac{dQ}{T}$, the change of entropy for perfect gases is found by dividing equations (1) and (2) by the absolute temperature, T.

Hence $$d\phi=\frac{dQ}{T}=c_v\,\frac{dt}{T}+(c_p-c_v)\,\frac{dv}{v}\tag{3}$$

and $$d\phi=\frac{dQ}{T}=c_p\,\frac{d}{T}-(c_p-c_v)\,\frac{dp}{p}\tag{4}$$

4. *Constant Volume Lines.*—Integrating equation (3) between limits of ϕ and ϕ_1, T and T_1, and v and v_1,

$$\phi-\phi_1=c_v\log_e\frac{T}{T_1}+(c_p-c_v)\log_e\frac{v}{v_1}\tag{5}$$

If the volume is constant during the change of entropy, equation (5) reduces to

$$\phi-\phi_1=c_v\log_e\frac{T}{T_1}\tag{6}$$

In Fig. 1, equation (6) gives a logarithmitic curve, and in Fig. 2, plotting log T as ordinates, the equation gives a straight line. From equation (4) it is evident that successive constant volume lines are parallel on both diagrams. The various lines may be drawn with any arbitrarily assumed zero of entropy.

5. *Constant Pressure Lines.*—From equation (4) the change of entropy between finite limits, becomes

$$\phi-\phi_1=c_p\log_e\frac{T}{T_1}-(c_p-c_v)\log_e\frac{p}{p_1}\tag{7}$$

which, at constant pressure, becomes

$$\phi-\phi_1=c_p\log_e\frac{T}{T_1}\tag{8}$$

When plotted, equation (8) becomes a logarithmitic curve in Fig. 1,—whose slope is different from that of the constant volume lines,—and a straight line in Fig. 2.

The proper location of the initially plotted constant pressure line with reference to the previously drawn constant volume lines is determined through the aid of the characteristic equation

$$pv=RT$$

Having drawn the initial constant pressure line, successive lines are parallel thereto as shown by equation (7).

6. *Isothermal and Adiabatic Lines.*—By construction, horizontal lines on both diagrams are lines of constant temperature, or isothermals, and vertical lines are lines of constant entropy, or adiabatics.

7. *pv^n Lines.*—For air the equation of the adiabatic is

$$p_o v_o^k=pv^k=pv^{1.405}$$

and for the isothermal it is

$$p_o v_o=pv$$

In practice expansion or compression lines lie somewhere between these two extremes and the equation of the real curve becomes

$$p_o v_o^n = pv^n$$

where n lies between 1 and 1.405 for air.

Since $$pv = RT$$

$$pv^n = p_1 v_1^n$$

and $$Tv^{n-1} = T_1 v_1^{n-1}$$

then $$\frac{v}{v_1} = \left(\frac{T_1}{T}\right)^{\frac{1}{n-1}} \qquad (9)$$

Inserting this value of $\frac{v}{v_1}$ in equation (5)

$$\phi - \phi_1 = c_v log_e \frac{T}{T_1} + \frac{c_p - c_v}{n-1} log_e \frac{T}{T_1}$$

$$= c_v \frac{n-k}{n-1} log_e \frac{T}{T_1} \qquad (10)$$

Assuming an initial condition of pressure and temperature and a value for n, equation (10) may be plotted as a curve in Fig. 1 or a straight line in Fig. 2. A series of such lines with different values of n are shown on Fig. 2. Being more difficult to construct and to use when plotted on Fig. 1, these pv^n lines are omitted on this diagram.

8. *Intrinsic Energy.*—The intrinsic energy of the so-called perfect gases is in the form of sensible heat. Since the intrinsic energy is changed into mechanical energy, during an adiabatic expansion, the total intrinsic energy could be realized during an expansion to infinity. That is, the intrinsic energy in B. t. u. is

$$AE_1 = A\int_{v_1}^{\infty} pdv = A\frac{p_1 v_1}{k-1} = A\frac{RT_1}{k-1} \qquad (11)$$

where A is the reciprocal of the mechanical equivalent of heat, and is therefore equal to $\frac{1}{778}$. For air, equation (11) becomes

$$AE_1 = 0.16851\ T_1\ B.\ t.\ u. \qquad (12)$$

or $$E_1 = 131.1\ T_1 \text{ foot pounds} \qquad (13)$$

Since for air (or any other perfect gas) the intrinsic energy is thus a function of the absolute temperature, only, its values for the several temperatures have been calculated and indicated on the diagrams. Obviously all curves and figures given are for one pound of air, since the value of R used is based on this unit of weight.

From equations (11) and (12) it is evident that for air and other perfect gases, isothermals are lines of constant intrinsic energy or isodynamics.

9. *Variable Specific Heats.*—In the construction of both entropy-temperature diagrams, the specific heats c_p and c_v are assumed to be constant within the temperature range given. If these specific heats are considered to be variable, and if their values are assumed to be of the form

$$c_p = a + bT$$
$$c_v = a_1 + bT$$

the expressions for the change of entropy become respectively

$$\phi - \phi_1 = a_1 \log_e \frac{T}{T_1} + b(T - T_1) + (c_p - c_v) \log_e \frac{v}{v_1} \ldots\ldots\ldots (14)$$

$$= a \log_e \frac{T}{T_1} + b(T - T_1) - (c_p - c_v) \log_e \frac{p}{p_1} \ldots\ldots\ldots (15)$$

The constant volume, constant pressure, and pv^n lines from the above equations all become curves, whether plotted as in Fig. 1 or Fig. 2. Within the range of temperatures given on the charts, it is probable that the constant and generally used values of c_p and c_v are amply accurate.

III. The Use of the Diagrams

10. *Values of p, v, and T for Isothermal Changes.*—Values of p, v, and T for any condition of one pound of air may be read directly from the diagrams, Fig. 1 or Fig. 2. Since $pv = p_1v_1$ for the isothermal line, the final pressure or volume may be determined by following a constant temperature line through the given initial condition to the given final condition. Thus for example, find the initial and final volumes of one pound of air at 100° Fahr., which, starting at 100 lb. per sq. in. absolute, expands to 14.7 lb. absolute. From the diagram 1, the intersection of the 100 lb. and 100° lines is at 2.08 cu. ft.; and the intersection of the 100° and 14.7 lb. lines is at 14 cu. ft. By calculation the initial volume is 2.07 cu. ft., giving an error of −0.01 cu. ft. or 0.48%; and the final volume is 14.08 cu. ft., giving an error of −0.08 cu. ft. or 0.57%.

11. *Values of p, v, and T for Adiabatic Changes.*—For the adiabatic, the following relations hold:

$$p_1v_1^k = pv^k \ldots\ldots\ldots (16)$$

$$T_1v_1^{k-1} = Tv^{k-1} \ldots\ldots\ldots (17)$$

$$T_1p_1^{\frac{1-k}{k}} = Tp^{\frac{1-k}{k}} \ldots\ldots\ldots (18)$$

These relations are determined graphically by following the vertical adiabatic or isentropic line through the given initial and final conditions. For example, let the initial absolute pressure and volume be respectively 100 lb. per sq. in. and 4 cu. ft., and the final pressure, 14.7 lb.

absolute. From the diagram the 100 lb. and 4 foot lines intersect at 625° Fahr. Following a vertical line through this point, the final temperature is found to be 162° Fahr. and the final volume 15.6 cu. ft. By calculation, the initial temperature is 621° Fahr. showing the error due to the use of the chart to be $+4°$ or 0.37%; the final temperature is 165° Fahr., giving an error of $+3°$ or 0.5%; and the final volume is 15.66 cu. ft., giving an error of $+0.06$ cu. ft. or 0.38%.

12. *Values of p, v, and T for pv^n Changes.*—If k be replaced by n, the formulas just given hold for changes along the pv^n lines. Fig. 2 permits the graphical relation of the several quantities to be determined in a manner similar to that just described for the adiabatics, except that a line is drawn through the given initial condition, parallel to the line having the desired value of n, and the results read at the final condition.

13. *Heat Changes.*—In general the heat change is expressed by the equation

$$dQ = A(dE + pdv)$$

or

$$Q = A\left(E_1 - E_2 + \int_{v_2}^{v_1} pdv\right) \qquad (19)$$

where Q = the heat added or rejected in *B. t. u.*

E_1 and E_2 = the initial and final intrinsic energy in foot pounds.

A = the reciprocal of the mechanical equivalent of heat.

$\int_{v_2}^{v_1} pdv$ = the external work in foot pounds done during the change Q.

14. *Graphical Representation of Heat Changes.*—On the diagram, Fig. 1, areas are quantities of heat in *B. t. u.* according to the scale of the drawing shown. The total heat change in passing from one condition to another on the diagram, is the area under the line or curve representing the path of the air during the change, to absolute zero. Since the lowest temperature shown is $-200°$ Fahr. (259.5° absolute), the area under the given path to $-200°$ must be increased by the addition of an amount of heat equal to 259.5 times the total change of entropy between the initial and final conditions of the air, to find the value Q.

15. *Values for External Work.*—Since $A(E_1 - E_2)$ can be read directly from the diagram the external work becomes

$$A\int_{v_2}^{v_1} pdv = Q - A(E_1 - E_2) \text{ in } B.\ t.\ u. \qquad (20)$$

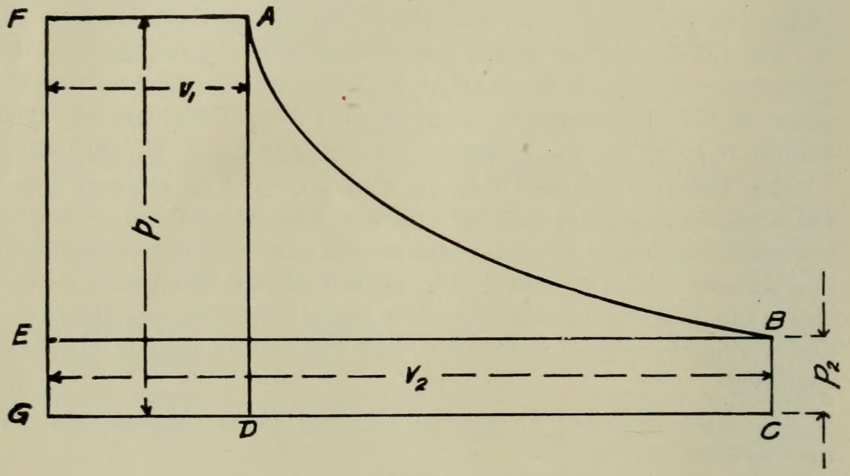

FIG. 3

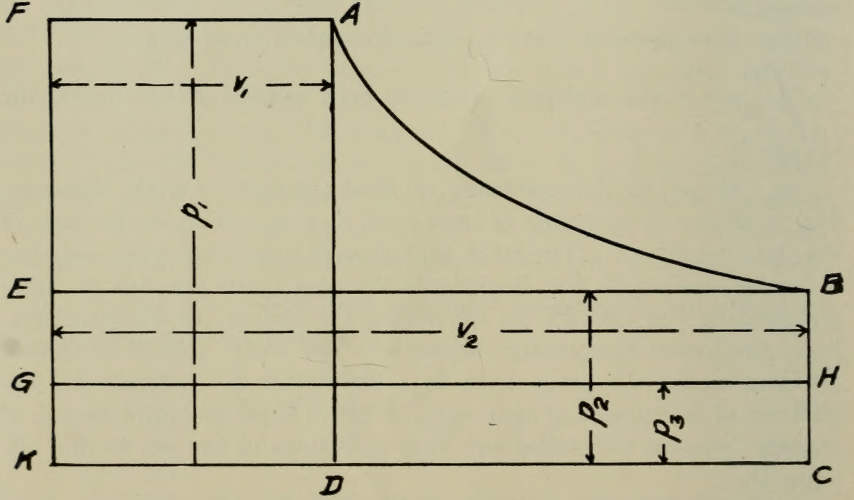

FIG. 4

16. *Expansion or Compression Along Irregular Paths.*—If for any reason air is expanded or compressed along an irregular path, and the changes of pressure and volume are known, the several pressures and volumes (reduced to a basis of one pound weight) may be directly plotted on Fig. 1, and the change in heat determined by mechanical integration of the area under the curve, as described in the preceding article.

The method here presented for finding Q offers no particular advantage, since the work done may be obtained by mechanical integration of the pv diagram, and the change of intrinsic energy, calculated or taken from the diagram, added to the work done to secure the value of Q, as indicated by equation (19).

17. *Isothermal Expansion and Compression.*—The work done in *B. t. u.* during an isothermal change is

$$W = Ap_1v_1 \log_e \frac{v_2}{v_1} = Ap_1v_1 \log_e \frac{p_1}{p_2}$$

$$= ART_1 \log_e \frac{p_1}{p_2} \text{ in } B.\ t.\ u. \qquad (21)$$

In these formulas p_1 and v_1 are the pressure and volume at A in Fig. 3 and 4, and p_2 and v_2 are the pressure and volume at B.

On either Fig. 1 or Fig. 2 the heat change in *B. t. u.* can be obtained for an isothermal expansion or compression by multiplying the absolute temperature (459.5°+ temperature Fahr. on the chart) by the change in entropy between the given initial and final condition of the gas. Inspection of the diagram shows that the change of intrinsic energy is 0 during an isothermal change, hence the heat added or rejected is exactly equal to the external work done, expressed in *B. t. u.*

The work thus obtained is equivalent to that obtained under the curves AB in Fig. 3 and 4, represented by the areas $ABCD$.

The work of a complete cycle of admission, expansion, and exhaust, or of suction, compression, and discharge as shown in Fig. 3 by the area $ABEF$ is exactly equal to the work done under the curve AB (area $ABCD$) since $p_1v_1 = p_2v_2$. In Fig. 4, with incomplete expansion, the area $ABCD$ = area $ABEF$, hence the total work done in *B. t. u.* is equal to area $ABCD$, found from the entropy-temperature diagram, increased by the addition of the area $EBHG$, or $A(p_2 - p_3)v_2$.

It must be remembered that in these formulas pressures are in pounds per square foot and volumes in cubic feet.

To illustrate the application of the diagrams let it be required to find the work done during an expansion of one pound of air from 100 lb. per sq. in. absolute to 14.7 lb. absolute at 100° Fahr. From the diagram the initial and final values for the entropy are 0.2316 and 0.1, or the entropy change is 0.1316. Hence the work in *B. t. u.* is

$$\begin{aligned} W &= 0.1316\ (100+459.5) \\ &= 73.63 \end{aligned}$$

By calculation, $W = 73.43$ *B. t. u.*, so the error is $+0.2$ *B. t. u.* or 0.27%

18. *Adiabatic Expansion or Compression.*—The work done during an adiabatic expansion, represented by the areas *ABCD* in Figs. 3 and 4, and expressed in *B. t. u.*, may be found analytically by one of the following formulas:

$$W = \frac{Ap_1v_1}{k-1}\left[1-\left(\frac{v_1}{v_2}\right)^{k-1}\right] \tag{22}$$

$$= \frac{Ap_1v_1}{k-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right] \tag{23}$$

$$= \frac{A\ (p_1v_1-p_2v_2)}{k-1} \tag{24}$$

$$= AR\ \frac{(T_1-T_2)}{k-1} \tag{25}$$

Similarly, the work done during an adiabatic compression, represented by the area *ABCD* in Fig. 3 and expressed in *B. t. u.*, may be found by the use of any one of the following formulas:

$$W = \frac{Ap_2v_2}{k-1}\left[\left(\frac{v_2}{v_1}\right)^{k-1}-1\right] \tag{26}$$

$$= \frac{Ap_2v_2}{k-1}\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}-1\right] \tag{27}$$

$$= \frac{A(p_1v_1-p_2v_2)}{k-1} \tag{28}$$

$$= \frac{AR(T_1-T_2)}{k-1} \tag{29}$$

Through the aid of the charts, the use of these somewhat troublesome formulas is avoided, since the work in *B. t. u.* is found by determining the difference between the initial and final intrinsic energy along an adiabatic (isentropic) or vertical line through the initial and final states of the gas.

For example, if one pound of air expands from an initial pressure and volume of 100 lb. per sq. in. absolute and 4 cu. ft., respectively, to 14.7 lb. per sq. in. absolute, the change of intrinsic energy (and hence the work) is 78.03 *B. t. u.* Through the aid of equation (23) the work is calculated to be 77.56 *B. t. u.*, giving an error of +0.47 *B. t. u.* or 0.66%.

19. *The Work of the Complete Cycle.*—If the work of a complete cycle of admission, expansion to back pressure, and exhaust; or of suction, compression, and discharge is required for a motor or compressor without clearance as shown by area *ABEF* in Fig. 3, it can be determined analytically as follows:

Area *ABEF* = Area *ABCD* + Area *ADGF* − Area *BCGE* or, using formulas (24) or (28).

$$Ap_1v_1+\frac{A(p_1v_1-p_2v_2)}{k-1}-Ap_2v_2=\frac{Ak\ (p_1v_1-p_2v_2)}{k-1} \quad \ldots\ldots\ldots\ldots\ldots (30)$$

That is, the area of the cycle *ABEF* in Fig. 3 is equal to the absolute work done under the curve *AB* multiplied by k, the value of which for air = 1.405. Evidently the change in intrinsic energy multiplied by k gives the net work of the cycle under the conditions here stated.

If a motor does not expand the air to the back pressure line, as in Fig. 4, the area $BHGE = A(P_2-P_3)V_2$ must be added to k times the change of intrinsic energy between A and B to secure the work of the cycle, *ABHGF*.

20. *Expansion or Compression Along a pv^n Line.*—If k be replaced by n in the equations for adiabatic expansion and compression, equations (22) to (30) inclusive, the work done under a pv^n curve is obtained. Taking equation (25) or (29) and replacing k by n the work done becomes, in *B. t. u.*,

$$W=\frac{AR(T_1-T_2)}{n-1}=\frac{k-1}{n-1}\cdot\frac{AR(T_1-T_2)}{k-1} \quad \ldots\ldots\ldots\ldots (31)$$

From this last value for W in equation (31) it is evident that the work done during an adiabatic change between T_1 and T_2 needs only to be multiplied by $\frac{k-1}{n-1}$ to secure the corresponding work done during a pv^n change.

Similarly to the adiabatic cycle, the work of the complete pv^n cycle can be found by multiplying the work done during expansion or compression by n (See Fig. 3).

TABLE 1

Values of n	Values of $\frac{k-1}{n-1}$
1.05	8.10
1.10	4.05
1.15	2.70
1.20	2.025
1.25	1.62
1.28	1.446
1.30	1.35
1.32	1.266
1.35	1.157
1.38	1.066
1.405	1.000

21. *Heat Changes Along a pv^n Line.*—Since there must be a change in the value of Q along a pv^n curve, the heat added or rejected may be determined by the algebraic addition of the change in intrinsic energy between the initial and final conditions and the work done during the change. If the heat change is required without regard to the work done, it may be found as above and reduced to the simplest terms, as follows:

$$Q_{12} = \mp\frac{AE(T_1-T_2)}{k-1} \pm \frac{k-1}{n-1} \cdot \frac{AR(T_1-T_2)}{k-1}$$

$$= \pm\frac{k-n}{n-1} \cdot \frac{AR(T_1-T_2)}{k-1} \qquad \text{.......................} (32)$$

Evidently the heat change is found by multiplying the change of intrinsic energy from the diagram by the factor $\frac{k-n}{n-1}$. Table 2 gives values of this factor.

TABLE 2

Values of n	Values of $\frac{k-n}{n-1}$
1.05	7.10
1.10	3.05
1.15	1.70
1.20	1.025
1.25	0.62
1.28	0.446
1.30	0.35
1.32	0.266
1.35	0.157
1.38	0.066
1.405	0.000

To show the method of handling the pv^n lines on the chart, the following problem is solved:

Given $p_1 = 150$ lb. per sq. in. absolute
$p_2 = 14.7$ lb. per sq. in. absolute
$t_1 = 200°$ Fahr.

to determine the initial and final volumes, the final temperature, the work done during expansion in *B. t. u.*, the heat added in *B. t. u.*, and the work of the complete cycle, all per pound of air, if $pv^{1.2} = C$.

Following a line through the given initial and final conditions, parallel to the $pv^{1.2}$ line it is found from the diagram that $V_1 = 1.613$

cu. ft., and by calculation $V_1=1.625$, an error of -0.012 cu. ft. or 0.7%; from the diagram $V_2=11.48$ cu. ft., and by calculation $V_1=11.25$ cu. ft., an error of $+0.23$ cu. ft. or 2%; from the diagram $T_2=-12°$ Fahr., and by calculation $T_1=11.7°$. The change of intrinsic energy is found to be 35.724 *B. t. u.* from the diagram. From Table 1, for $n=1.2$ the factor for work $=2.025$. Hence $35.724\times2.025=72.34$ *B. t. u.* is the external work under the expansion curve. By calculation the value is found to be 72.39 *B. t. u.* The work of the cycle is

$$72.34\times1.2=86.81 \text{ } B.\ t.\ u.$$

while the heat change is

$$+(35.724\times1.025)=+36.617 \text{ } B.\ t.\ u. \text{ (See Table 2).}$$

As is evident, the agreement between the values from the chart with those calculated is amply close for all practical purposes.

22. *Flow of Air Through Nozzles.*—It is easily shown that if air flows through a short nozzle the heat energy changed to kinetic energy per pound of air is, in *B. t. u.*,

$$\frac{k}{k-1}.A(p_1v_1-p_2v_2)=A\ \frac{u^2}{2g}\ldots\ldots\ldots\ldots\ldots\ldots(33)$$

Where $u=$ the velocity of flow in feet per second. The left hand member of the above equation is recognized as the work of the cycle *ABEF* shown in Fig. 3, the determination of which has been discussed in article 19. Remembering that $A=\frac{1}{778}$ and $2g=64.4$, the value of u can be easily calculated.

It must be remembered that the pressure, p_2, at the point of minimum diameter of the nozzle has a value such that

$$p_2=0.5767\ p_1$$

when the nozzle has a well rounded entrance and the pressure p_1 is more than twice the pressure in the space into which the nozzle is discharging. If the nozzle is properly flared beyond the point of minimum diameter the air will be further expanded with a resultant increase in velocity of flow. The minimum diameter and the relation between p_2 and p_1 determine the actual weight or volume of free air discharged.

The velocity resulting from a given expenditure of heat is plotted graphically on the "Mollier Diagram" accompanying Marks & Davis' "Steam Tables." If, therefore, the heat converted into kinetic energy is found graphically from Figs. 1 or 2 or analytically from equation (33) the graphical relation referred to enables the velocity to be determined without further calculation. In the design of nozzles, these graphical methods save a large amount of time while giving a satisfactory degree of accuracy.

IV. Use of the Charts for Gases Other Than Air

Although the entropy-temperature diagrams here shown are calculated for one pound of air, it is possible to use them for other gases through the aid of corrective factors.

In Table 3 the gases listed each have almost exactly the same value of k, and the values of the constant R vary directly as the relative specific volumes. In consequence, the equation $\frac{pv}{T}=R$ for air must be multiplied by the relative specific volume of another gas. Thus, for example, if the pressure and volume of one pound of hydrogen are the same as on the air diagram, the *absolute* temperature will be 14.46 times as great as the absolute air temperature; or if the pressure and temperature of hydrogen are the same as on the air diagram, the volume will be 14.46 times as great as the indicated volume of air. Further, it is evident that the change of intrinsic energy between two points on the air diagram will be 14.46 times as great for H as for air, since k is the same as for air. Or, stating the case in another way, *the diagrams are exactly accurate for* $\frac{1}{14.46}$ *pounds of hydrogen.* In a similar way the corrective factors may be employed for the other gases given.

Table 3

$k=1.4+$

Kind of Gas	Relative Density	Relative Specific Volume	Correction Factor
H	1	14.46	14.46
O	16	0.904	0.904
N	14	1.03	1.03
CO	14	1.03	1.03
Air	14.46	1.00	1.00

For gases whose values of k are different from that for air, a correction factor is needed in determining the relations p, v, and T, and another factor for determining the relations between the work done in comparison with air.

The relation between the work done during an adiabatic change between two temperatures for any gas compared with air is

$$\text{Work per pound of gas} = \frac{R'\ (k-1)}{R\ (k'-1)}\ (\text{Work per pound of air})$$

where R' and k' are the values of these constants for the given gas. Table 4 shows the two correction factors for CO_2, CH_4 and C_2H_4.

Mixed gases whose properties are known may be treated in the same manner as for the three gases mentioned. The necessity for using these correction factors renders this application of the entropy-temperature charts of doubtful value.

TABLE 4

Kind of Gas	Value of k	Relative Density $H=1$	Relative Specific Volume	Correction Factor for p, v, and T	Correction Factor for Adiabatic work $\frac{R'(k-1)}{R(k'-1)}$
CH_4	1.267	8	1.808	1.808	2.73
C_2H_4	1.213	14	1.03	1.03	1.96
CO	1.29	22	0.657	0.657	0.919

V. THE AIR FLOW DIAGRAM

The application of the principles of thermodynamics to the flow of air in pipes leads to the following formula connecting the several variables entering into the problem:

$$u_1 = \sqrt{\frac{gRTm}{KL} \cdot \frac{p_1^2 - p_2^2}{p_1^2}} \quad \text{.......... (35)}$$

where u_1 = the initial velocity of flow in feet per second.

g = the acceleration due to gravity.

T = the absolute temperature, considered constant.

m = the "hydraulic mean depth," or the ratio of the area to the perimeter of the pipe.

K = the coefficient of resistance.

L = the length of the pipe in feet.

p_1 and p_2 = the initial and final pressures in the pipe in pounds per square inch, absolute.

If V = the number of cubic feet of free air at 70° Fahr. and 14.7 pounds pressure flowing per minute.

d = the *actual* diameter of pipe in inches.

$r = \frac{p_2}{p_1}$ = the ratio of the final to the initial pressures.

$m = \frac{d}{4}$ for circular pipes.

then equation (35) reduces to

$$V = 3.061\sqrt{\frac{d^5 p^2_1\,(1-r^2)}{KL}} \quad \text{.......... (36)}$$

In plotting the chart (Fig. 5) the value of K has been assumed to be

$$K = 0.003\left(1 + \frac{3.6}{d}\right)$$

and, instead of plotting r direct

$100\left(\frac{p_1 - p_2}{p_1}\right)$ has been used as more convenient, since it shows the percentage of pressure drop in the pipe in terms of the initial pressure.

While the formula for V is theoretically correct, its ultimate accuracy depends upon the assumption of a correct value of K, which is not easy considering the paucity of reliable data available.

Should it be desired to use K', another value for K than the one given, the values of V from the chart are to be multiplied by $\sqrt{\frac{K}{K'}}$

Should the discharge from a pipe be required for another temperature than 70° Fahr. (530° absolute) the values of V from the chart are to be multiplied by $\sqrt{\frac{T'}{530}}$, where T' is the absolute temperature desired.

The use of the air flow diagram is simple. If the volume of free air per minute, the length of pipe, the initial pressure, and the permissible drop in pressure are given, proceed as follows:

From the length of pipe in feet, follow a vertical line to its intersection with the volume line; from this point follow a horizontal line to its intersection with the initial pressure line, and from there follow a vertical line until it intersects the horizontal line through the given pressure drop. The position of this last point determines the proper pipe diameter. Any other combination of given data is handled similarly.

This diagram may be adapted to other gases than air (assuming the coefficients of resistance to be the same) by multiplying the volume of air by $\sqrt{\frac{v'}{v}}$ where v^1 and v are respectively the specific volumes of the given gas and of air; or by $\sqrt{\frac{1}{D}}$ where D is the density of the given gas in terms of air.

To illustrate, let $L=2000$ feet, $V=1000$ cu. ft., $p_1=100$ lb., drop $=5\%$. Following a vertical line from $L=2000$ to $V=1000$, a horizontal line to $p=100$, a vertical line to drop $=5\%$, it is found that a 4-inch pipe is the nearest commercial size available, although the drop for this size is about 6%.

Without the aid of the chart, the solution of the problem just given would require considerable time since the pipe diameter can only be obtained from the formula by a series of approximations, and at least three trial solutions would ordinarily be necessary.

VI. Maximum Power Transmitted Through Pipes

The maximum power which may be secured per pound of air is realized during a complete cycle when the expansion is complete to the back pressure line, that is to atmospheric pressure, as shown by Fig. 3.

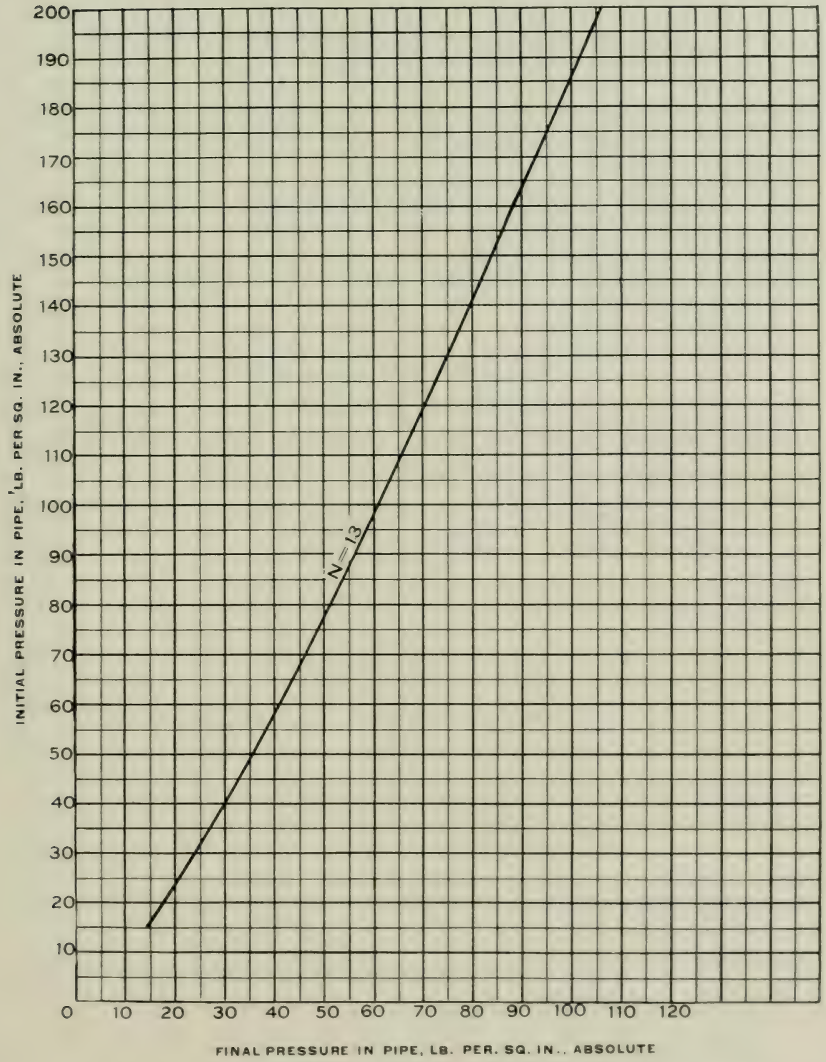

FIG. 6. MAXIMUM POWER TRANSMITTED BY AIR, $N = 1.3$

When air is transmitted through pipes there is a loss of pressure due to friction, but, in general, there will be no loss of temperature; hence the product of the pressure and volume remains constant. Because of this fact, the loss of *power* is less than the loss of *pressure*, assuming that the air may be effectively used at the reduced pressure. Obviously, the power developed per cycle is a function of the value of n for the expansion curve, so its probable value is necessarily known in determining the actual efficiency of transmission.

Let p_1 = the absolute initial pressure in the pipe.
p_2 = the absolute final pressure in the pipe.
p_a = the atmospheric pressure.

The work developed per cycle with no loss of pressure, is, in foot pounds,

$$W_1 = \frac{n}{n-1} p_1 v_1 \left[1 - \left(\frac{p_a}{p_1} \right)^{\frac{n-1}{n}} \right] \quad \ldots\ldots\ldots\ldots\ldots (37)$$

Where the pressure is reduced to p_2 during the transmission the maximum possible work developed becomes

$$W_2 = \frac{n}{n-1} p_1 v_1 \left[1 - \left(\frac{p_a}{p_2} \right)^{\frac{n-1}{n}} \right[\quad \ldots\ldots\ldots\ldots\ldots (38)$$

The efficiency of power transmission is then

$$E_1 = \frac{W_2}{W_1} = \frac{1 - \left(\frac{p_a}{p_2} \right)^{\frac{n-1}{n}}}{1 - \left(\frac{p_a}{p_2} \right)^{\frac{n-1}{n}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

while the efficiency of *pressure transmission* is

$$E_2 = \frac{p_2}{p_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Equation (38) gives the power theoretically available at the end of a main of given diameter and length if v_1 is replaced by V the volume of air transmitted per minute. Inserting the value of V from equation (36) in equation (38) gives

$$W = 3.061 \frac{n}{n-1} \left[\frac{d^5 p_1^2 \, (1 - r^2)}{KL} \right]^{\frac{1}{2}} \left[1 - \left(\frac{p_a}{p_2} \right)^{\frac{n-1}{n}} \right]$$

Since $r^2 = \left(\frac{p_2}{p_1} \right)^2$

$$W = 3.061 \frac{n}{n-1} \left[\frac{d^5 \, (p_1^2 - p_2^2)}{KL} \right]^{\frac{1}{2}} \left[1 - \left(\frac{p_a}{p_2} \right)^{\frac{n-1}{n}} \right] \ldots\ldots\ldots\ldots\ldots (41)$$

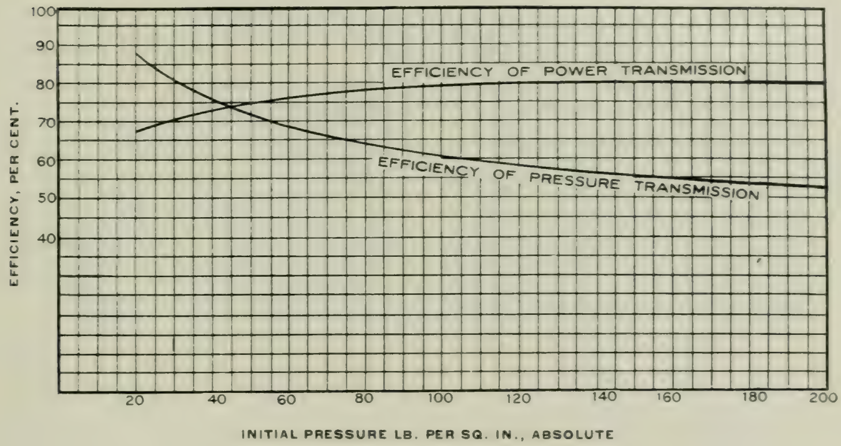

FIG. 7. EFFICIENCY OF POWER TRANSMISSION FOR MAXIMUM POWER, $N = 1.3$

Since W and p_2 are the only variables in equation (41), if W be differentiated with respect to p_2 and the result placed equal to 0, it is found that W becomes a maximum when

$$p_1^2 = \frac{p_2^2}{n-1}\left[n\left(\frac{p_2}{p_a}\right)^{\frac{n-1}{n}} - 1\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Solving equation (42) for various values of n from 1.2 to 1.4, it is found that the influence of n affects but slightly the relation between p_1 and p_2. Since in practice the real value of n is probably not far from 1.3, this value is used in plotting the curves shown Fig. 6 and 7.

In Fig. 6, values of p_1 are plotted as ordinates and of p_2 as abscissas so the final pressure in the pipe for theoretical maximum power transmitted can be determined directly for initial pressures up to 200 pounds per square inch absolute. Fig. 7 shows the efficiency of *power* transmission, calculated from equation (39), and the efficiency of *pressure* transmission, calculated from equation (40), when the power transmitted is a maximum with the final pressures as shown in Fig. 6 for $n = 1.3$. As previously stated other values of n between 1.2 and 1.4 give but little deviation from the results shown in Fig. 6 and 7.

From this discussion it is evident that there is a theoretical maximum carrying power for an air pipe line. Thus with air at 100 pounds initial pressure, the power transmitted is a maximum when the final pressure is 61 pounds absolute. The efficiency of *pressure* transmission is then 61%, while the efficiency of *power* transmission is nearly 79%.

This latter efficiency cannot be secured if the air is used non-expansively or with only partial expansion, nor can the maximum power be attained under such conditions.

While this subject is probably of greater theoretical than practical interest it is of value in determining the minimum size of pipe which should be used for a given service. It is possible in temporary installations that the reduced cost of the pipe, carrying its maximum power, would more than offset the loss in the transmission. Under normal conditions, however, this would not be true, and the pipe line should be proportioned to give not more than 5% to 10% pressure drop.

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